MATH-8 Sample Test 4

100 points NAME: Solns

Show all work neatly. EXACT answers unless specified.

(1) Given the vectors **u** = 2 **i** + 2 **j** and **v** = -4**i** + 3**j** , find the following:

a) $\|\mathbf{u}\| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$ $\quad$ $2\sqrt{2}$

b) 3**u** + **v** $\quad \langle 6,6\rangle + \langle -4,3\rangle$ $\quad$ $\langle 2,9\rangle$

c) **u** • **v** $\quad 2(-4)+2(3)$ $\quad$ -2

d) The angle between **u** and **v** $\quad$ $\cos^{-1}\left(\frac{-1}{5\sqrt{2}}\right) \approx 98^\circ$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{-2}{2\sqrt{2}(5)} = \frac{-1}{5\sqrt{2}}$$

e) The direction angle of **v** (exact) $\quad$ $\tan^{-1}\left(\frac{-3}{4}\right)+180^\circ \approx 143.1^\circ$

$$\tan\theta = \frac{-3}{4} \text{ in Q2}$$

f) Find a value for b such that < b,2> is orthogonal to **v** $\quad$ $b = +3/2$

$$\langle b,2\rangle \cdot \langle -4,3\rangle = 0$$
$$-4b+6=0$$

g) Find a unit vector in the direction of **v** $\quad$ $\left\langle -\frac{4}{5}, \frac{3}{5}\right\rangle$

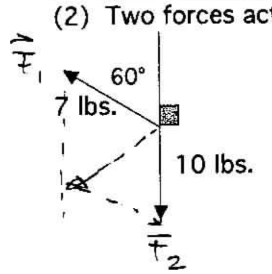
$$\frac{1}{\|\vec{v}\|}\vec{v}$$

h) If **PQ** is a representative of **v** where P=(3,-1), find the coordinates of point Q. $(-1,2)$

Let $Q=(x_1,x_2)$ then $\vec{PQ} = \langle x_1-3, x_2+1\rangle = \langle -4,3\rangle \Rightarrow x_1-3=-4 \quad x_2+1=3$

$x_1=-1 \quad x_2=2$

(2) Two forces act on an object as shown. Find the magnitude and the direction of the resultant.



(exact and approx.) (10 pts)

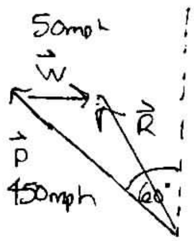
$$\vec{F}_1 = \langle 7\cos 150^\circ, 7\sin 150^\circ\rangle = \left\langle -\frac{7\sqrt{3}}{2}, \frac{7}{2}\right\rangle$$
$$\vec{F}_2 = \langle 0,-10\rangle$$

$$\vec{R} = \vec{F}_1+\vec{F}_2 = \left\langle -\frac{7\sqrt{3}}{2}, -\frac{13}{2}\right\rangle$$

$$\|\vec{R}\| = \sqrt{\left(-\frac{7\sqrt{3}}{2}\right)^2 + \left(-\frac{13}{2}\right)^2} = \sqrt{79} \approx 8.9 \text{ lbs}$$

$$\tan\theta = \frac{-13/2}{-7\sqrt{3}/2} = \frac{13}{7\sqrt{3}} \text{ in Q3} \qquad \theta = \tan^{-1}\left(\frac{13}{7\sqrt{3}}\right)+180^\circ \approx 227^\circ$$

(3) An airplane is traveling at a constant airspeed of 450 mph in the direction N60°W. If wind is blowing directly eastward at a rate of 50 mph, what is the actual speed and direction of the airplane?



$\vec{p} = \langle 450\cos 150°, 450\sin 150° \rangle = \langle -225\sqrt{3}, 225 \rangle$

$\vec{w} = \langle 50, 0 \rangle$

$\vec{R} = \vec{p} + \vec{w} = \langle 50 - 225\sqrt{3}, 225 \rangle$

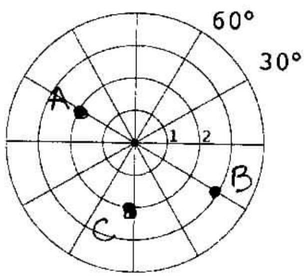
$\|\vec{R}\| = \sqrt{(50-225\sqrt{3})^2 + (225)^2} \approx 407$ mph

$\tan\theta = \frac{225}{50-225\sqrt{3}}$ in Q2

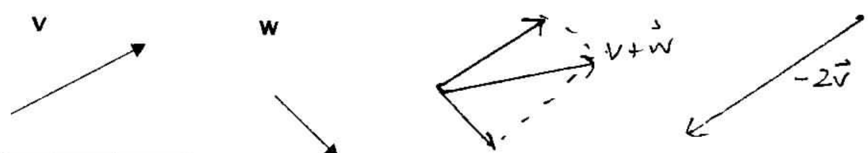
$\theta = 180° + \tan^{-1}\left(\frac{225}{50-225\sqrt{3}}\right)$

$\approx 146.5°$

(4) On the axes below, plot (and label) the polar points A(2, 150°), B(3, -π/6) , C(-2, π/2) (3pts)



(5) Given the vectors **w** and **v** below, find **w + v** and **-2v** .



(6) Given the point (5, 7π/4) in polar coordinates, find the rectangular representation.

$x = r\cos\theta = 5\cos\frac{7\pi}{4}$

$y = r\sin\theta = 5\sin\frac{7\pi}{4}$

$\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$

(7) Given the point $\left(-1, \sqrt{3}\right)$ in rectangular coordinates, find two different polar representations; one with r > 0, the other with r < 0.

$r^2 = (-1)^2 + \sqrt{3}^2 = 4$

$r = \pm 2$

$\tan\theta = -\sqrt{3}$

Q2

$\left(2, \frac{2\pi}{3}\right)$

$\left(-2, \frac{5\pi}{3}\right)$

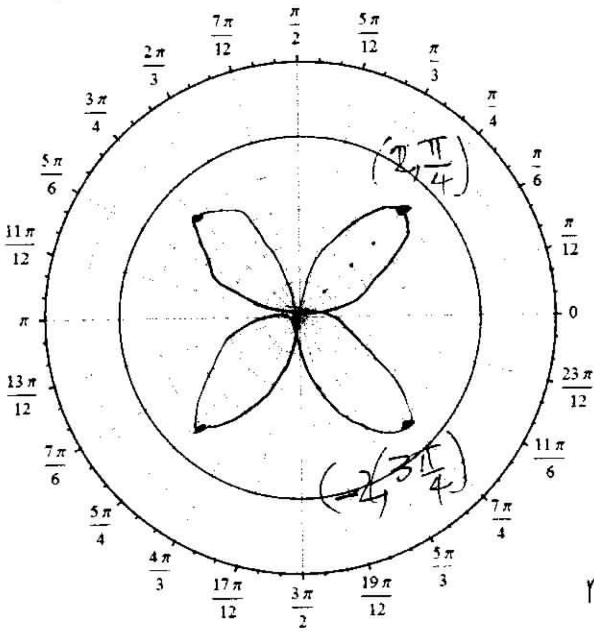
(8) Convert to rectangular coordinates: $r = \cos\theta + \sin\theta$

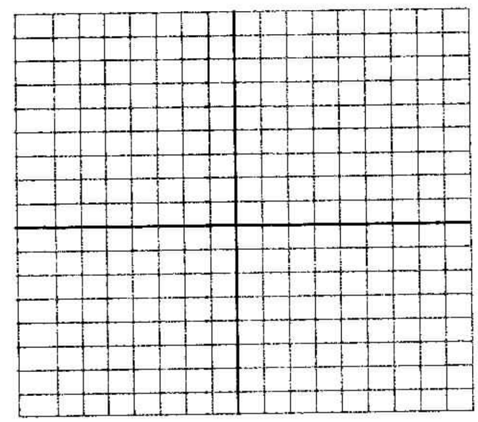
mult. by r

$r^2 = r\cos\theta + r\sin\theta$

$x^2 + y^2 = x + y$

(9) Graph the polar curve: $r=4\sin 2\theta$. (You may use either grid)





$r = 4\sin 2\theta \Rightarrow$ Rose 4 petals

spacing $\frac{2\pi}{4} = \frac{\pi}{2}$

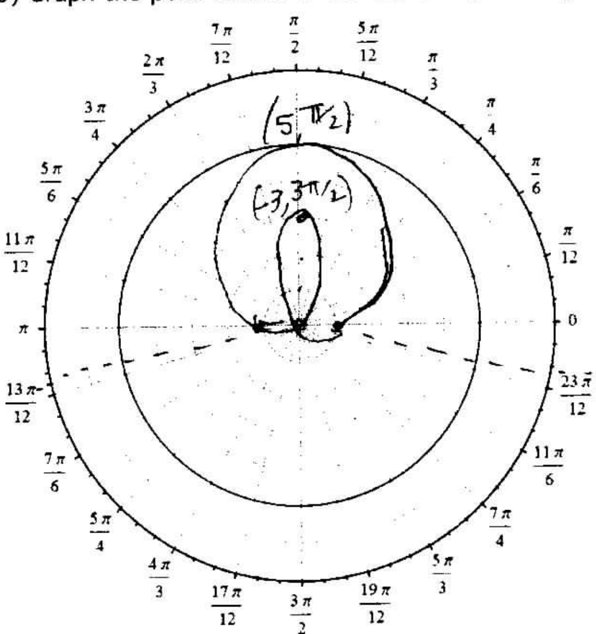
tip when $r=4$ $\quad 4 = 4\sin 2\theta$

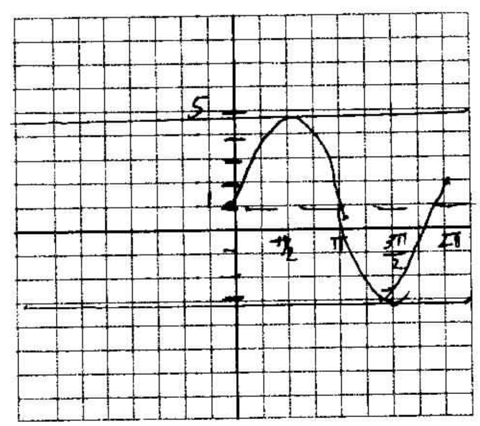
$\sin 2\theta = 1$

$2\theta = \frac{\pi}{2}$

$\theta = \pi/4$

(10) Graph the polar curve: $r=1+4\sin\theta$. (You may use either grid)





$r=0$

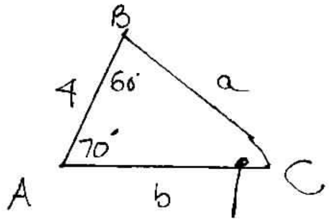
$1+4\sin\theta=0$

$\sin\theta = -1/4$

$\pi + \sin^{-1}\frac{1}{4}$ $\qquad 2\pi - \sin^{-1}(-\frac{1}{4})$

3.4 $\qquad$ 6.03

(11) Find all remaining parts of the following triangle(s) $c = 4$, $B = 60^\circ$, $A = 70^\circ$, and find the area. Approx. accurately (i.e. used "stored values") to one decimal place.



$$\frac{b}{\sin 60^\circ} = \frac{a}{\sin 70^\circ} = \frac{4}{\sin 50^\circ}$$

$C = 180^\circ - 70^\circ - 60^\circ$

$a \approx$ 4.9

$b \approx$ 4.5

$C \approx$ 50°

$Area \approx$ 8.5

Area

$\frac{1}{2} bc \sin A$

$\frac{1}{2} b \cdot 4 \sin 70^\circ$

$2b \sin 70^\circ$

$$\frac{b}{\sin 60^\circ} = \frac{4}{\sin 50^\circ} \qquad \frac{a}{\sin 70^\circ} = \frac{4}{\sin 50^\circ}$$

$$b = \frac{4 \sin 60^\circ}{\sin 50^\circ} = \frac{2\sqrt{3}}{\sin 50^\circ} \approx 4.5 \qquad a = \frac{4 \sin 70^\circ}{\sin 50^\circ} \approx 4.9$$

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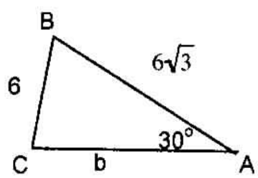
(12) Find all remaining parts of the given triangle(s), $\boxed{\text{exactly}}$.



$$\frac{\sin B}{b} = \frac{\sin 30^\circ}{6} = \frac{\sin C}{6\sqrt{3}}$$

$$\sin C = \frac{6\sqrt{3} \sin 30^\circ}{6} = \frac{\sqrt{3}}{2}$$

$C = 60^\circ, 120^\circ$ Two Triangles

$\boxed{C_1 = 60^\circ \quad B_1 = 90^\circ}$

$$\frac{\sin B}{b} = \frac{\sin 30^\circ}{6}$$

$$b_1 = \frac{6 \sin B_1}{\sin 30^\circ} = 12 \sin B_1 = 12$$

$\boxed{C_2 = 120^\circ \quad B_2 = 30^\circ}$

$$b_2 = 12 \sin B_2 = 12 \cdot \frac{1}{2} = \boxed{6}$$

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(13) Evaluate each of the following exactly:

This topic was on the last test, not here. Replace with trig equation

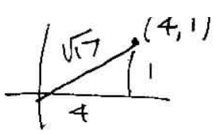
(a) $\cos(\tan^{-1}(1/4)) =$ $4/\sqrt{17}$

$^{-1}(-3/4)) =$ $\frac{-\sqrt{7}}{3}$

$\theta = \tan^{-1} \frac{1}{4}$

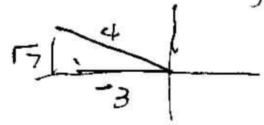
$\tan\theta = \frac{1}{4}$, Q1

$\cos\theta = \frac{4}{\sqrt{17}}$



$\theta = \cos^{-1}(-3/4)$

$\cos\theta = -3/4$, Q2

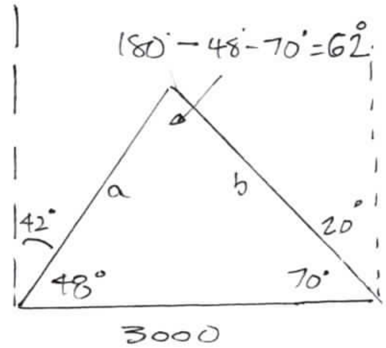


$\tan\theta = -\frac{\sqrt{7}}{3}$

(14)

The Colonel spots a campfire at a of bearing N42°E from his current position. Sarge, who is positioned 3000 feet due east of the Colonel, reckons the bearing to the fire to be N20°W from his current position. Determine the distance from the campfire to each man, rounded to the nearest foot.

$$\frac{a}{\sin 70^\circ} = \frac{b}{\sin 48^\circ} = \frac{3000}{\sin 62^\circ}$$



$$a = \frac{3000\sin 70^\circ}{\sin 62^\circ} \qquad b = \frac{3000\sin 48^\circ}{\sin 62^\circ}$$

$a \approx 3193$ ft $\qquad b \approx 2525$ ft

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(15)

Time Lost to a Navigation Error In attempting to fly from city P to city Q, an aircraft followed a course that was 10° in error, as indicated in the figure. After flying a distance of 50 miles, the pilot corrected the course by turning at point R and flying 300 miles farther. If the constant speed of the aircraft was 250 miles per hour, how much time was lost due to the error?

$$\frac{d}{\sin R} = \frac{300}{\sin 10^\circ} = \frac{50}{\sin Q}$$

$$300\sin Q = 50\sin 10^\circ$$

$$\sin Q = \frac{50\sin 10^\circ}{300}$$

must be sm angle.

$$Q = \sin^{-1}\left(\frac{50\sin 10^\circ}{300}\right) \approx 1.7^\circ \quad (A)$$

$$R = 180^\circ - 10^\circ - (A) \approx 168.3^\circ \quad (B)$$

On correct path,

$\cancel{d^2 = 50^2 + 300^2 - 2(50)(300)\cos}$

$$d = \frac{300\sin R}{\sin 10^\circ} \approx 349.1 \quad (C)$$

$$t = \frac{\text{dist}}{\text{rate}} = \frac{d}{250} \approx 1.396 \text{ hrs.}$$

On path taken

$$t = \frac{d}{r} = \frac{350}{250} = 1.4 \text{ hrs}$$

It took ≈ 0.0035 hrs more $= 0.0035 \text{ hrs} \cdot \frac{60 \text{ min}}{\text{hr}}$

$\approx .21$ min ≈ 12.7 seconds